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International Journal of Fatigue xxx (2007) xxx–xxx

International  
Journal of  
Fatigue[www.elsevier.com/locate/ijfatigue](http://www.elsevier.com/locate/ijfatigue)

## The influence of grains' crystallographic orientations on advancing short crack

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Received 31 August 2006; received in revised form 16 January 2007; accepted 31 January 2007

### 8 Abstract

9 A model of microstructurally short cracks that accounts for random grain geometry and crystallographic orientations is coupled with  
 10 crystal plasticity constitutive model. A short crack is then inserted in the slip plane in one of the grains at the model top boundary and  
 11 extended into one of the available slip planes of the neighboring grain at monotonic remote load of  $0.96R_{p0.2}$ . Crack tip opening (CTOD)  
 12 and sliding (CTSD) displacements are then calculated for several different crystallographic orientations and crack lengths. As the crack is  
 13 contained in a single grain the crystallographic orientation of the neighboring grain can change the crack tip displacements by up to 26%,  
 14 however, the displacements change by up to a factor of 10, once the crack is extended beyond the grain boundary into the next grain.  
 15 Significant CTSD values were observed in all the analyzed cases pointing to mixed mode loading. Another important observation is that  
 16 the random crystallographic orientations of grains beyond the first two crack-containing grains affect the CTOD by a factor of up to 4.4.  
 17 This effect decreases slightly with increased crack length.  
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19 *Keywords:* Short cracks; Crystallographic orientations; Crack tip displacements; Polycrystalline material; Crystal plasticity  
 20

### 21 1. Introduction

22 Microstructurally short cracks behave differently from  
 23 the long ones. Their propagation rate and path is strongly  
 24 influenced by local microstructural features such as grain  
 25 boundaries, crystallographic orientations, inclusions, voids  
 26 and material phases, etc. [1,2]. Often initiated from persis-  
 27 tent slip bands these cracks propagate on the slip planes,  
 28 creating zigzag patterns [3–5] when changing the slip plane.  
 29 The crack tip loading is therefore generally mixed mode  
 30 with strong shear component. All these influence result in  
 31 a variable crack propagation rates. Vašek [6] for example  
 32 observed that crack propagation rates may vary signifi-  
 33 cantly for nominally identical cracks.

34 When creating models of such short cracks care should  
 35 be taken to incorporate mentioned microstructural fea-

36 tures. In recent years several authors applied crystal plas-  
 37 ticity material models [7–9] but without the explicit grain  
 38 shape modeling. Several works use rectangular grain  
 39 shapes [10,11]. Random grain geometry and crystal plastic-  
 40 ity have been used in [12] where calculated  $J$ -integral values  
 41 are compared with the isotropic case while in [13] scatter of  
 42 the  $J$ -integral values has been determined. Both works deal  
 43 with intergranular cracks.

44 In authors' previous studies a combination of random  
 45 grain geometry and crystal plasticity was used to study  
 46 the crack tip slip activity and the influence of formation  
 47 of shear bands on the crack tip displacements [14]. For a  
 48 bicrystal configuration the preferential slip plane was deter-  
 49 mined in [15]. Fixed crack length was used in both works.  
 50 The present study, however, investigates the influence of  
 51 crystallographic orientations on the crack tip displacements  
 52 for an advancing crack, including cases where the crack  
 53 passes the grain boundary. The relation between the crack  
 54 length, crack tip displacements and their spread due to  
 55 the random grain orientations is studied. Specifically, we

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56 want to see the way in which random grain orientations  
57 change the crack tip displacements with the increased crack  
58 length.

## 59 2. Model description

### 60 2.1. Constitutive model

61 Monocrystal's elastic behaviour is generally anisotropic  
62 and is governed by the generalized Hooke's law,  
63  $\sigma_{ij}^e = C_{ijkl} \epsilon_{kl}^e$ , where  $\sigma_{ij}^e$  represents the second-rank elastic  
64 stress tensor,  $C_{ijkl}$  the fourth-rank stiffness tensor and  $\epsilon_{kl}^e$   
65 the second-rank elastic strain tensor. The number of inde-  
66 pendent elastic constants for a cubic crystal system is 3.

67 The material's plastic behaviour at the grain level is  
68 modeled with crystal plasticity theory [16,17] with an over-  
69 view given below. Further details on the kinematics are  
70 given in [18]. The basic assumption is that the material  
71 flows through the crystal lattice via dislocation motion,  
72 while the lattice itself, with material embedded on it, under-  
73 goes elastic deformations and rotations. The plastic deforma-  
74 tion of a monocrystal is assumed to arise solely from  
75 simple shear on a specific set of slip planes, see Fig. 1 with  
76 projections given in Fig. 2. Deformation by other mecha-  
77 nisms such as for example diffusion, twinning and grain  
78 boundary sliding is currently not taken into the account.  
79 The total deformation gradient is

$$81 F_{ij} = F_{ik}^* \cdot F_{kj}^p, \quad (1)$$

82 where  $F_{ik}^*$  is the elastic part associated with stretching and  
83 rotation of the lattice while  $F_{kj}^p$  is the part of the total deforma-  
84 tion gradient due solely to slip. The velocity gradient in  
85 the current state is given by a standard formula

$$87 L_{ij} = \dot{F}_{ik} \cdot F_{kj}^{-1} = D_{ij} + \Omega_{ij} \quad (2)$$

88 and can be expressed as a sum of the symmetric rate of  
89 stretching  $D_{ij}$  and antisymmetric spin tensor  $\Omega_{ij}$ .  $D_{ij}$  and  $\Omega_{ij}$   
90 can be further decomposed into plastic parts ( $D_{ij}^p$  and  $\Omega_{ij}^p$ )  
91 and lattice or elastic parts ( $D_{ij}^*$  and  $\Omega_{ij}^*$ ), i.e.,  $D_{ij} = D_{ij}^* + D_{ij}^p$

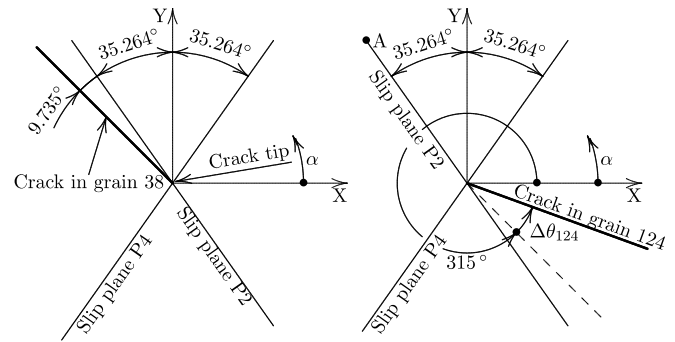


Fig. 2. Relation between the crack plane and slip planes P2 and P4 at crystallographic orientation of  $\alpha = 0^\circ$ . Left: crack in grain 38. Right: an example of a crack in grain 124.

and  $\Omega_{ij} = \Omega_{ij}^* + \Omega_{ij}^p$ . Now, the plastic part of the velocity gra- 92  
dient in the current state can be expressed as 93

$$D_{ij}^p + \Omega_{ij}^p = \sum_{\alpha} \dot{\gamma}^{(\alpha)} s_i^{(\alpha)} m_j^{(\alpha)}, \quad (3) \quad 94$$

where the summation is performed over all active slip sys- 97  
tems,  $(\alpha)$ , defined by their normal  $m_i^{(\alpha)}$  and a shearing direc- 98  
tion,  $s_i^{(\alpha)}$ .  $\dot{\gamma}^{(\alpha)}$  represents the shear rate. The cumulative slip, 99  
 $\gamma$ , is defined as, 100

$$\gamma = \sum_{\alpha} \int_0^t |\dot{\gamma}^{(\alpha)}| dt. \quad (4) \quad 102$$

Plastic strain rate is then obtained from the symmetric part 103  
of Eq. (3) 104

$$\dot{\epsilon}_{ij}^p = \sum_{\alpha} \frac{1}{2} \dot{\gamma}^{(\alpha)} (s_i^{(\alpha)} m_j^{(\alpha)} + s_j^{(\alpha)} m_i^{(\alpha)}). \quad (5) \quad 106$$

The constitutive relation of the elastic–plastic monocrystal 107  
is now given in terms of stress and strain rates as, 108  
 $\dot{\sigma}_{ij} = C_{ijkl} (\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^p)$  [19]. It is assumed that the shear rate 109  
 $\dot{\gamma}^{(\alpha)}$  depends on the stress only via the Schmid resolved 110  
shear stress, Eq. (6). This is a reasonable approximation 111  
at room temperature and for ordinary strain rates and 112  
pressures [19]. Yielding is then assumed to take place when 113

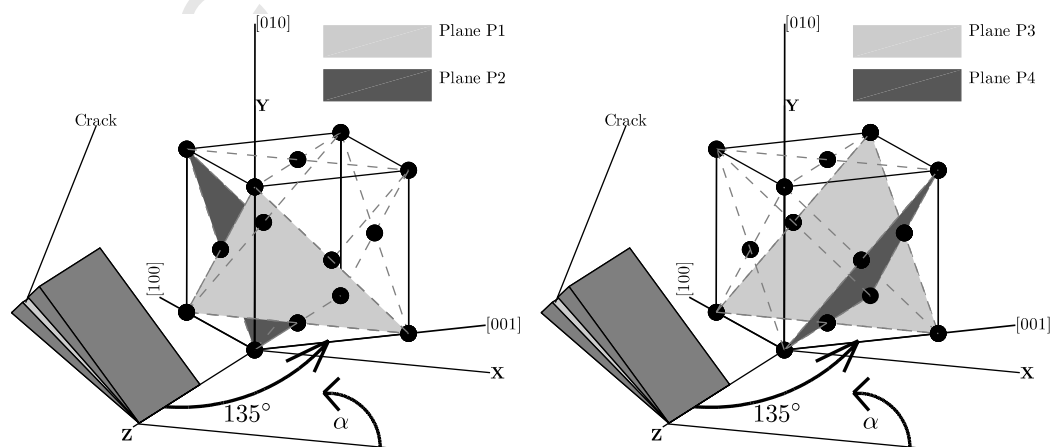


Fig. 1. Relation between the slip planes of a face centered cubic material and the crack for  $\alpha = 0^\circ$ . The crack plane is aligned with slip plane P2.

114 the Schmid resolved shear stress exceeds the critical shear  
 115 stress  $\tau_0$ .

118 
$$\dot{\gamma}^{(z)} = \dot{a}^{(z)} \left( \frac{\tau^{(z)}}{g^{(z)}} \right) \left| \frac{\tau^{(z)}}{g^{(z)}} \right|^{n-1}, \quad \tau^{(z)} = s_i^{(z)} \sigma_{ij} m_j^{(z)}, \quad (6)$$

119  $\dot{a}^{(z)}$  represents the reference strain rate,  $n$  is the strain rate-  
 120 sensitivity parameter and  $g^{(z)}$  is the current strain-hardened  
 121 state of the crystal. In the limit, as  $n$  approaches infinity, this  
 122 power law approaches that of a rate-independent material.  
 123 The current strain-hardened state  $g^{(z)}$  can be derived from,

125 
$$\dot{g}^{(z)} = \sum_{\beta} h_{z\beta} \dot{\gamma}^{(\beta)}, \quad (7)$$

126 where  $h_{z\beta}$  are the slip-hardening moduli defined by the  
 127 adopted hardening law. In this work Peirce et al. hardening  
 128 law is used [20], where self-hardening moduli  $h_{zz}$  are defined  
 129 by:

131 
$$h_{zz} = h(\gamma) = h_0 \operatorname{sech}^2 \left| \frac{h_0 \gamma}{\tau_s - \tau_0} \right|, \quad \operatorname{sech} = 1 / \cosh. \quad (8)$$

Here  $h_0$  stands for the initial hardening modulus,  $\tau_0$  is the  
 132 yield stress (equal to the initial value of the current strength  
 133  $g^{(z)}(0)$ ) and  $\tau_s$  a reference stress where large plastic flow initi-  
 134 ates. The latent-hardening moduli  $h_{z\beta}$  are determined as  
 135

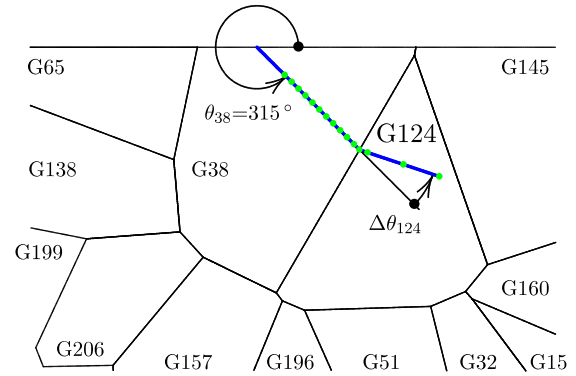


Fig. 4. Definitions of grain numbers, crack direction and deflection angles,  $\theta_{38}$  and  $\Delta\theta_{124}$ . Dots indicate most of the used crack lengths.

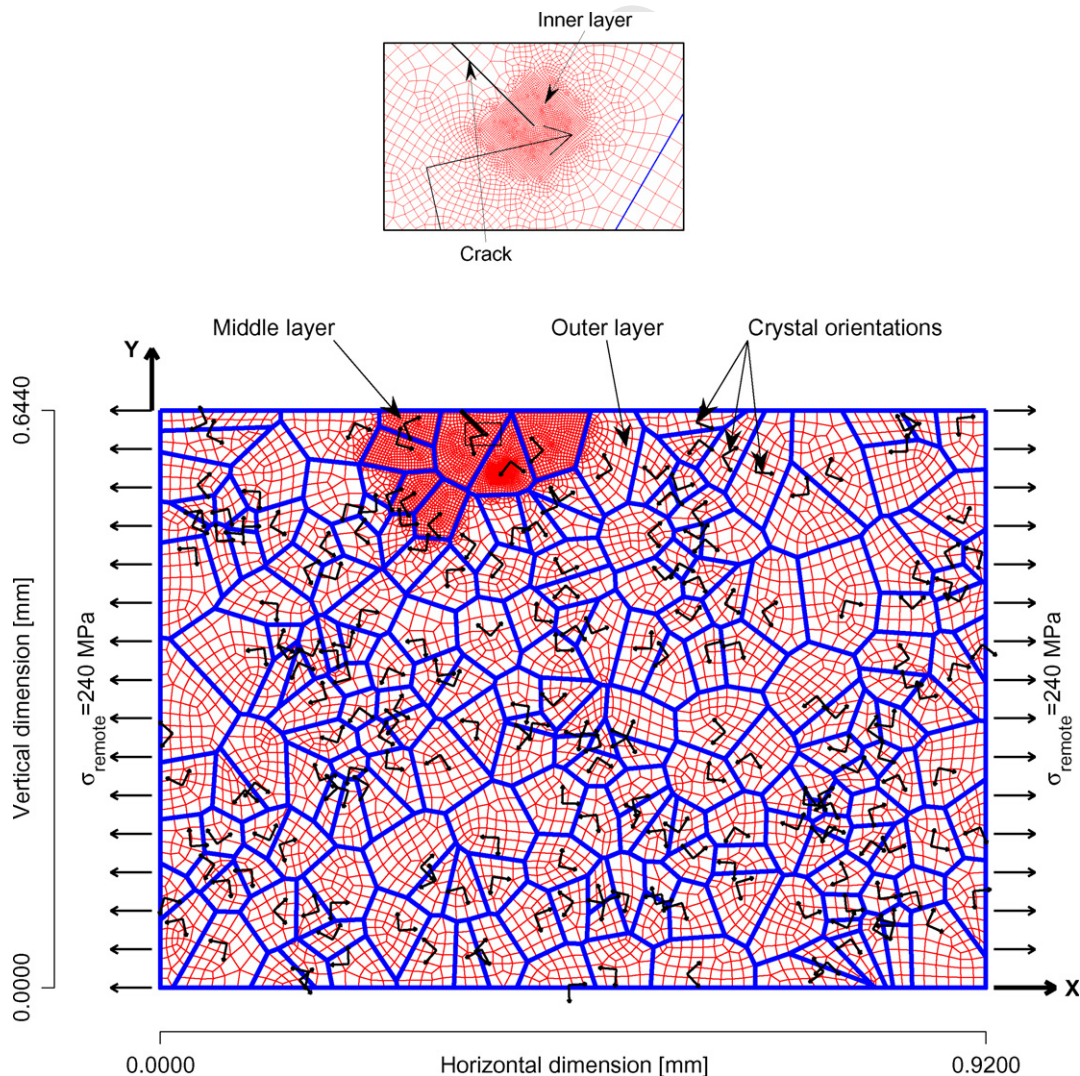


Fig. 3. The finite element model. Details of the crack tip mesh are shown in the insert.

136  $h_{\alpha\beta} = qh(\gamma)$ , ( $\alpha \neq \beta$ ), where  $q$  is a hardening factor. This  
 137 model was implemented as a user-subroutine into the finite  
 138 element code ABAQUS [21] and includes versions for small  
 139 deformation theory and rigorous theory of finite-strain and  
 140 finite-rotation. The latter was used in this work.

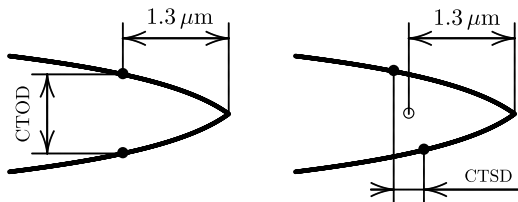


Fig. 5. Definition of the CTOD and CTSD.

## 2.2. Layout of structural model

141

The structural model is a planar rectangular aggregate  
 142 consisting of 212 randomly sized and shaped grains. The  
 143 aggregate is a planar Voronoi tessellation generated using  
 144 code VorTESS [22]. The model results in an average grain  
 145 size of about 53  $\mu\text{m}$  which is in agreement with published  
 146 values for 316L steel, reported to be between 50 and  
 147 80  $\mu\text{m}$ . Standard deviation of grain sizes divided by average  
 148 grain size is reported to be 0.32  $\mu\text{m}/\mu\text{m}$  [23]. 316L has relatively  
 149 weak morphological texture, with elongation of  
 150 grains in the rolling direction of about 20% [24]. This effect  
 151 was, however, not accounted for in our model. 316L is an  
 152 austenitic steel with a face centered body structure and is  
 153 used in certain nuclear power plant piping systems.  
 154

The finite element model of the grain structure with a  
 155 crack is presented in Fig. 3. Each grain is subdivided into  
 156 8-noded, reduced-integration, plane strain finite elements.  
 157 The mesh is composed out of inner, middle and outer layer.  
 158 The inner layer is defined by a rectangle of size  $10 \times 10 \mu\text{m}$ ,  
 159 with crack tip at its center. Average element size in the  
 160 inner layer of 0.125  $\mu\text{m}$  is achieved by assigning 80 elements  
 161 along each of the rectangle's sides and 40 elements  
 162 along the part of the crack pertaining to the inner layer.  
 163 Crack-containing grain (except the area of the inner layer)  
 164 and all the grains that have a common border with the  
 165 crack-containing grains compose the middle layer. Average  
 166 element size in this layer is 2.5  $\mu\text{m}$ . Grains beyond the  
 167

Table 1  
 Crystallographic orientations for grain 124 and corresponding crack  
 deflection angles

Crystallographic orientation		Crack deflection angle $\Delta\theta_{124}$	
$\alpha_{38}$	$\alpha_{124}$	Crack in SP2	Crack in SP4
9.735°	36.264°	26.528°	-44°
9.735°	56.264°	46.528°	-24°
9.735°	64.264°	54.528°	-16°
9.735°	70.264°	60.528°	-10°
9.735°	80°	70.264°	-0.264°

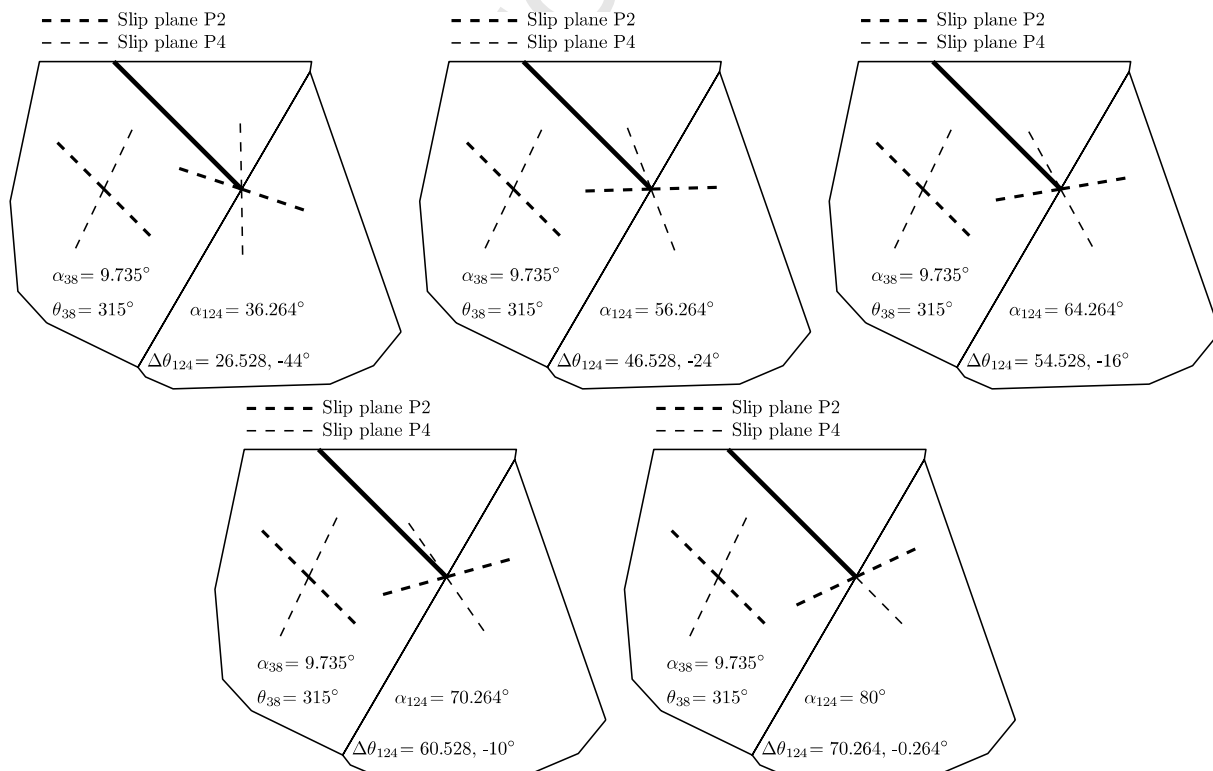


Fig. 6. Applied crystallographic orientations of grains 38 (9.735°) and 124 (36.264°, 56.264°, 64.264°, 70.264° and 80°). Two possible crack extension deflections are indicated with two values of  $\Delta\theta_{124}$ .

168 middle layer compose the outer layer. Average element size  
169 in this layer is 13  $\mu\text{m}$ . The finite element meshing of the  
170 grains in this layer is automatic and follows procedures  
171 outlined in [25].

172 To avoid violations of finite element shape constraints,  
173 only a subset of “meshable” Voronoi tessellations has been  
174 considered in the analysis. The “meshable” tessellations  
175 basically assume reasonably small aspect ratios of cord  
176 lengths. Further details are available in [25]. Such approach  
177 essentially prevents use of tessellations with very small  
178 grains.

179 Each grain is assumed to behave as a monocrystal gov-  
180 erned by the anisotropic elasticity and crystal plasticity  
181 models as described in the previous section. The number  
182 of grains included in the model is not sufficient to result  
183 in a size-independent macroscopic response of the aggre-  
184 gate (representative volume element). The overall global  
185 response of the aggregate will therefore still slightly depend  
186 on the applied set of crystallographic orientations and  
187 grain sizes and shapes. Should the aggregate be larger than  
188 the representative volume element, its overall response  
189 would not depend on the applied set of crystallographic  
190 orientations or grain sizes and shapes. However, the expe-  
191 rience with similar simulations shows that the error caused  
192 by this omission is limited to about 5% [26].

### 193 2.2.1. The crack and crystallographic orientations

194 A short inclined surface crack is introduced in the model.  
195 Crack is placed in a slip plane to mimic Stage I fatigue crack.  
196 This is achieved by first setting the angle between the crystal-  
197 lographic [100] direction and the macroscopic  $X$ -axis of all  
198 crystals in the model to  $135^\circ$  as shown in Fig. 1. The projec-  
199 tions of slip planes are given in Fig. 2. This results in a planar  
200 slip system model compatible with the planar macroscopic  
201 model. Next, each crystal is rotated around the global  $Z$ -axis  
202 by an angle  $\alpha$ , obtained by a random generator with uniform  
203 distribution. We will refer to this angle as crystallographic  
204 orientation. As reported in [24] 316L steel has a fairly strong  
205 crystallographic texture with a great number of grains hav-  
206 ing normal direction close to  $\langle 111 \rangle$  and  $\langle 100 \rangle$  direction.  
207  $\langle 100 \rangle$  direction is less pronounced while only a few grains  
208 exhibit a normal close to the  $\langle 110 \rangle$  direction. This texture  
209 was not taken into account because: (a) our intention is to  
210 maximize the scatter of the results due to crystallographic  
211 orientations, including the texture would decrease the scatter  
212 and (b) the effect of the texture is considered small compared  
213 to the 2D approximation of the aggregate used in this paper.

214 Crystallographic orientation of the grain 38 (the first  
215 crack-containing grain, see Fig. 4 for grain numbers)  
216 is set to  $9.375^\circ$  so that the crack falls into the slip plane P2,  
217 left-hand side of Fig. 2. For the face centered cubic material  
218 the angle between the slip planes P2 and P4 is  
219  $2 \times 35.264^\circ$ . Crystallographic orientation of the grain 124  
220 (second crack-containing grain) has to be such that the  
221 crack is either in slip plane P2 or P4. Crack direction in  
222 grain 124 is  $315^\circ + \Delta\theta_{124}$ , where  $\Delta\theta_{124}$  is crack deflection  
223 angle. Positive direction of  $\Delta\theta_{124}$  is in counter clockwise

224 direction. For the crack to fall into the slip plane P2, we  
225 select to rotate the crystal in the counter clockwise direc-  
226 tion until the point A on the slip plane P2 falls onto the  
227 crack, see right-hand side of Fig. 2. This condition is sat-  
228 isfied when  $315^\circ + \Delta\theta_{124} = \alpha_{124P2} + 90^\circ + 35.264^\circ + 180^\circ$ .  
229 Required crystallographic orientation of grain 124, for  
230 the crack to fall into the slip plane P2, is then given by  
231 Eq. (9). Similar arguments can be applied to obtain the  
232 crystallographic orientation of grain 124 for the crack to  
233 fall into the slip plane P4, Eq. (10).  
234

$$\alpha_{124P2} = (315^\circ + \Delta\theta_{124}) - 180^\circ - 90^\circ - 35.264^\circ, \quad (9)$$

$$\alpha_{124P4} = (315^\circ + \Delta\theta_{124}) - 180^\circ - 90^\circ + 35.264^\circ. \quad (10) \quad 236$$

237 The crack tip opening (CTOD) and sliding (CTSD) 237  
238 displacements are calculated at a distance of 2.5% of the  
239 average grain size behind the crack tip (i.e.  $0.025 \times 52.9 =$   
240  $1.3 \mu\text{m}$ ), see Fig. 5. This is consistent with examples found  
241 in the literature [11,27].

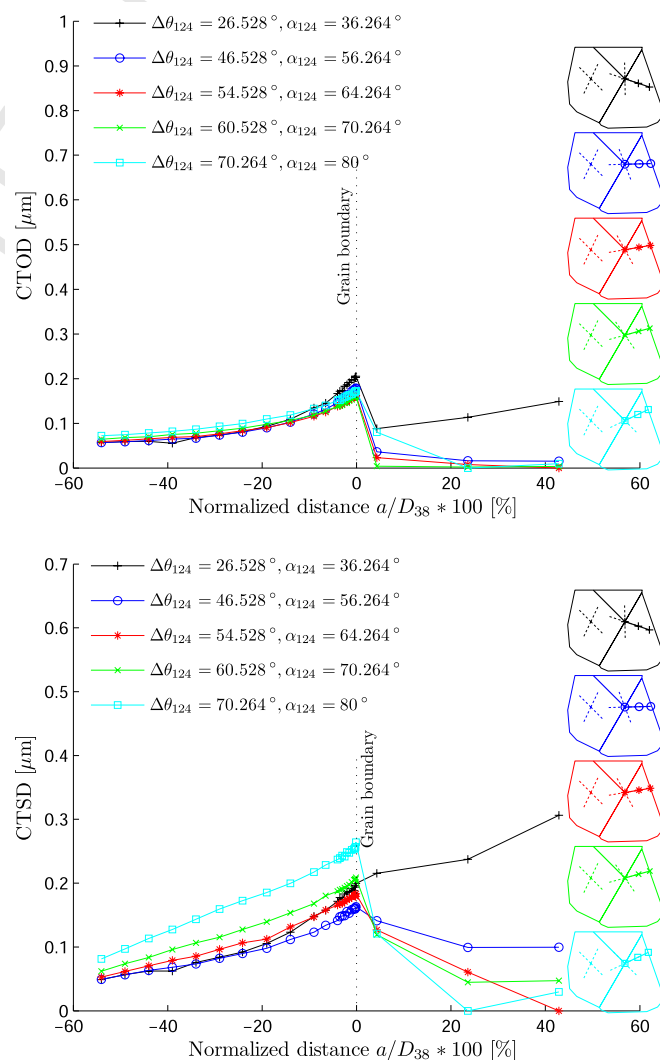


Fig. 7. The influence of different crystallographic orientations of grain 124 on the crack tip displacements. Crack in grain 124 is placed in slip plane P2.

Extensive mesh sensitivity study was performed in [28] resulting in the optimal mesh shown in Fig. 3. The applied mesh is expected to underestimate CTOD and CTSD by about 4.6% and 8.4% [28], respectively, which is deemed sufficient for the presented analysis.

### 2.2.2. Loading and boundary conditions

The applied macroscopic loading and boundary conditions are illustrated in Fig. 3. The left and right edges are loaded in macroscopic monotonic uniaxial tension up to a maximum load of  $0.96R_{p0.2}$  (240 MPa) with zero shear traction. This load is sufficient to trigger slip systems activity in all cases analyzed. The upper and lower edges are traction free. Prevention of rigid body movement is also imposed.

### 2.2.3. Material parameters

Elastic constants for AISI 316L single crystal were taken from literature [29]:  $C_{iiii} = 163,680$  MPa,  $C_{ijij} = 110,160$  MPa,  $C_{ijij} = 100,960$  MPa. Crystal plasticity

parameters are also taken from [29] and are as follows:  $h_0 = 330$  MPa,  $\tau_s = 270$  MPa,  $\tau_0 = 90$  MPa,  $n = 55$ ,  $q = 1.0$  and  $\dot{a}^{(\alpha)} = 0.001$ . The material parameters in [29] were obtained by fitting the macroscopic response of a polycrystalline model (macroscopic equivalent stress  $\langle \sigma_{ij} \rangle$  and strain  $\langle \epsilon_{ij} \rangle$ , estimated using volume averaging) to the measured true stress–strain curve of a polycrystalline specimen. Mean values correspond quite well with the experimental data. The two times standard deviation of the stress values at specific strains is on average 21 and 16.2 MPa for different grain geometries and crystallographic orientations, respectively.

## 3. Results

### 3.1. Variable crack lengths

In this section we examine crack tip displacements and accumulated slip around the crack tip as the crack is extended from the first into the second grain. The sizes of

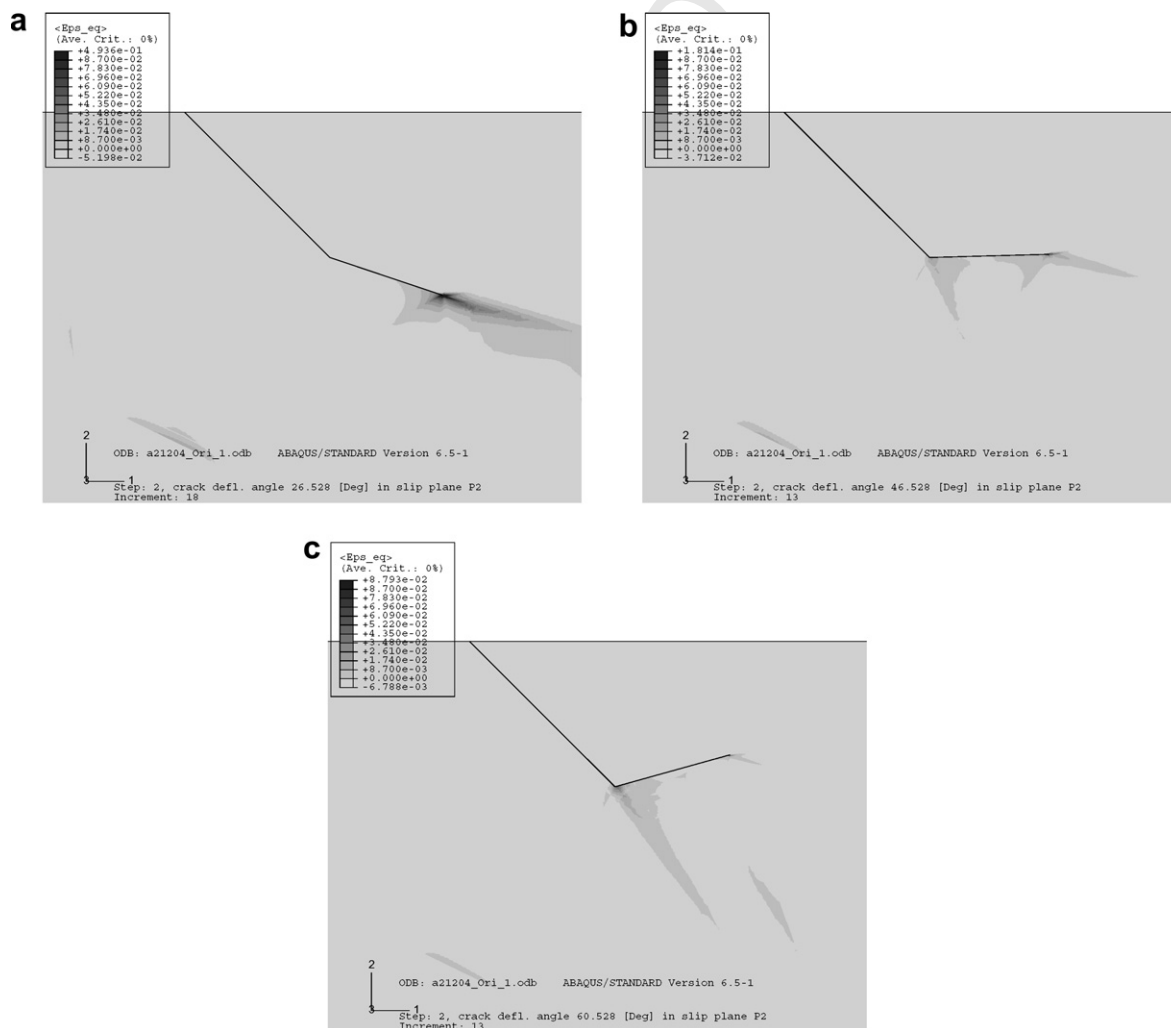


Fig. 8. Increasing the crack deflection angle results in a shift of position of maximal equivalent strain  $\langle \epsilon_{eq} \rangle$  from the crack tip point to the crack kink point. Crack in grain 124 is placed in slip plane P2.

277 these two grains can influence the crack tip displacements.  
 278 A larger grain with a favourable orientation will tend to  
 279 increase the crack tip displacements compared to a smaller  
 280 grain with the same orientation [14]. In all cases we assume  
 281 a Stage I crack, which propagates only through slip planes.  
 282 The crystallographic orientation of the grain 38 is rotated  
 283 in anti-clockwise direction by an angle  $\alpha = 9.735^\circ$  so that  
 284 the crack at an angle of  $\theta_{38} = 135^\circ$  falls onto the slip plane  
 285 P2. Let  $D_{38} = 70.87 \mu\text{m}$  stand for the size of the grain 38,  
 286 estimated as a square root of its area. We created a series  
 287 of models with embedded stationary cracks of different  
 288 lengths. Crack length in the grain 38 varies from  $0.25D_{38}$   
 289 up to  $0.739D_{38}$  which is almost on the grain boundary.  
 290 Once the crack extends across the grain boundary (into  
 291 the grain 124) its length is up to  $0.5D_{124}$ .  $D_{124} = 60.78 \mu\text{m}$   
 292 is the size of the grain 124, estimated as a square root of  
 293 its area. Several crystallographic orientations of the grain  
 294 124 are used while placing the crack in either slip plane  
 295 P2 or P4, see Table 1. Fig. 6 shows the corresponding

296 crystallographic orientations. The orientations of all other  
 297 grains are random.

298 Fig. 7 shows the CTOD and CTSD displacements for  
 299 crack in grain 124 placed in the slip plane P2. Abscissa's  
 300 value of 0 indicates the grain boundary. One can see that  
 301 different crystallographic orientations of grain 124 change  
 302 CTOD values by 26% for the case with the shortest crack.  
 303 As the crack is extended into the grain 124 this effect  
 304 becomes much more pronounced. This is to be expected  
 305 since the crack has to change its direction at the grain  
 306 boundary. It was observed that when the crack direction  
 307 turns upward to follow the slip plane P2, the maximal  
 308 equivalent strain (Mises equivalent strain),  $\langle \epsilon_{eq} \rangle$ , gradually  
 309 shifts from the crack tip to the crack kink point, see Fig. 8.  
 310 This results in up to 10 times smaller crack tip displacements  
 311 as the crack crosses the grain boundary. Another  
 312 important observation is that in all analyzed cases significant  
 313 CTSD values were observed. Cracks are therefore of  
 314 mixed mode loading.

315 Fig. 9 shows crack tip displacements when the crack in  
 316 grain 124 is placed in the slip plane P4. In these cases the  
 317 crack in grain 124 can be almost perpendicular to the external  
 318 load. In fact, we see that the closer the crack extension  
 319 (in the grain 124) is to being perpendicular to the external  
 320 load, larger the CTODs and lower the CTSDs are. This  
 321 would suggest that among all the available slip planes the  
 322 crack would probably propagate through the slip plane  
 323 that is more perpendicular to the external load. This is also  
 324 in line with well known observation where small crack  
 325 gradually transitions from Stage I to Stage II where its  
 326 direction is perpendicular to the external load.

327 Some models of microstructurally short cracks assume  
 328 that the crack propagation rate is associated with the  
 329 amount of accumulated plastic displacement along the slip  
 330 system in front of the crack tip [30]. We therefore calculated  
 331 the cumulative slip,  $\gamma$ , for elements within the inner

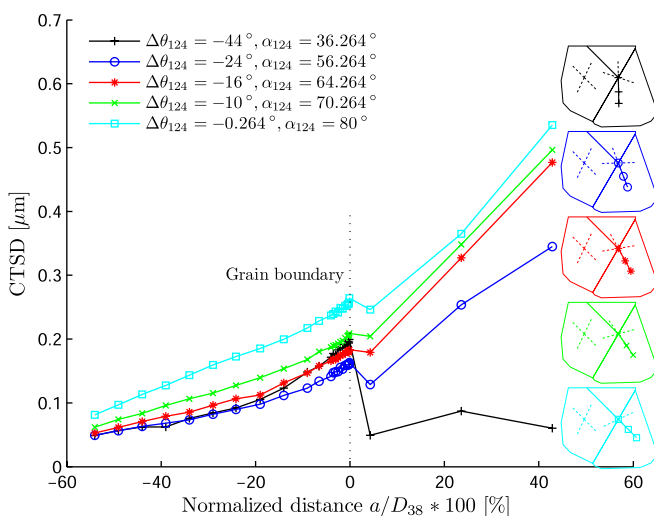
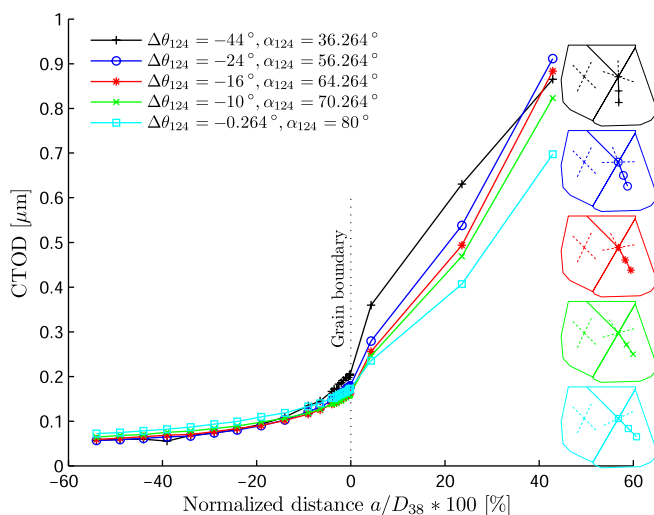


Fig. 9. The influence of different crystallographic orientations of grain 124 on the crack tip displacements. Crack in grain 124 is placed in slip plane P4.

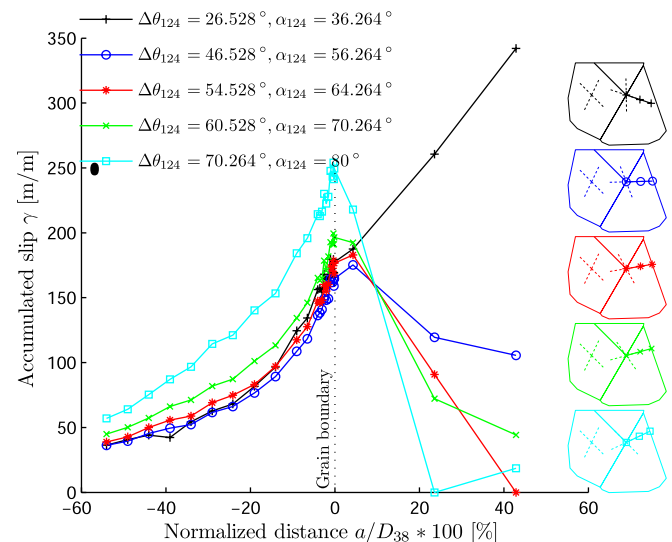


Fig. 10. Cumulative slip around the crack tip. Crack in grain 124 is placed in slip plane P2.

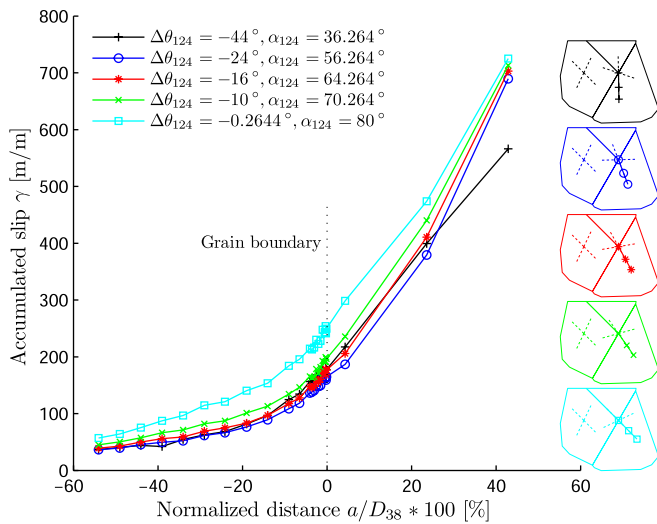


Fig. 11. Cumulative slip around the crack tip. Crack in grain 124 is placed in slip plane P4.

layer of elements around the crack tip, see the insert of Fig. 3. Figs. 10 and 11 show the accumulated slip when placing the crack in the grain 124 in either slip plane P2 or P4. The accumulated slip steadily increases until the crack comes into the vicinity of grain boundary. Beyond the grain boundary the orientation of the crack (defined by the crystallographic orientation) is very important for the accumulated slip. For a crack in slip plane P2, the accumulated slip decreases since the crack turns upwards, away from being perpendicular to the external load. Opposite is true for a crack in slip plane P4. One can see fairly good correlation of the accumulated slip with the crack tip displacements, suggesting that we could formulate a crack propagation criterion based on CTOD as well.

### 3.2. The influence of random crystallographic orientations 347

To evaluate the influence of random crystallographic orientations on the CTOD we generated 100 cases where for each fixed orientation of grains 38 ( $9.735^\circ$ ) and 124 ( $36.264^\circ$ ,  $56.264^\circ$ ,  $64.264^\circ$ ,  $70.264^\circ$  and  $80^\circ$ ) all other grains were randomly oriented. This was done for two different crack lengths: (a) crack located entirely in the grain 38 with crack length  $0.5D_{38}$  and (b) crack extended up to half grain 124 size ( $0.5D_{124}$ ). Crack in grain 124 was placed in slip plane P4. For each case a cumulative probability (distribution) function was calculated. Cumulative probability function, e.g.  $F(x)$ , is defined as a probability that an observed value (in our case calculated CTOD value) is less than or equal to  $x$ . The corresponding results are presented in Figs. 12 and 13. For a crack located entirely in grain 38 we see that different lines are very close to each other. This suggests that the orientation of the grain 124 has a relative small effect on CTOD values when the crack is contained in the first cracked grain (38). However, the scatter of the results along the abscissa shows that the orientations of grains beyond grains 38 and 124 have a significant impact. Changing the orientations of these grains resulted in a scatter of CTOD values by a factor of 4.4.

The impact of the grain's 124 orientation, however, increases once the crack is extended into the grain 124. This can be deduced from a larger distance between different lines in Fig. 13. On the other hand, the impact of (orientations of) grains beyond grains 38 and 124 on CTOD values decreases, manifesting itself in decreased scatter of values along abscissa-3.3 compared to 4.4 for a previous case. Two interlinked factors should also be mentioned once the crack is extended into the grain 124. The first is crack deflection. Larger CTODs are obtained when the crack extension in grain 124 is more perpendicular to the external

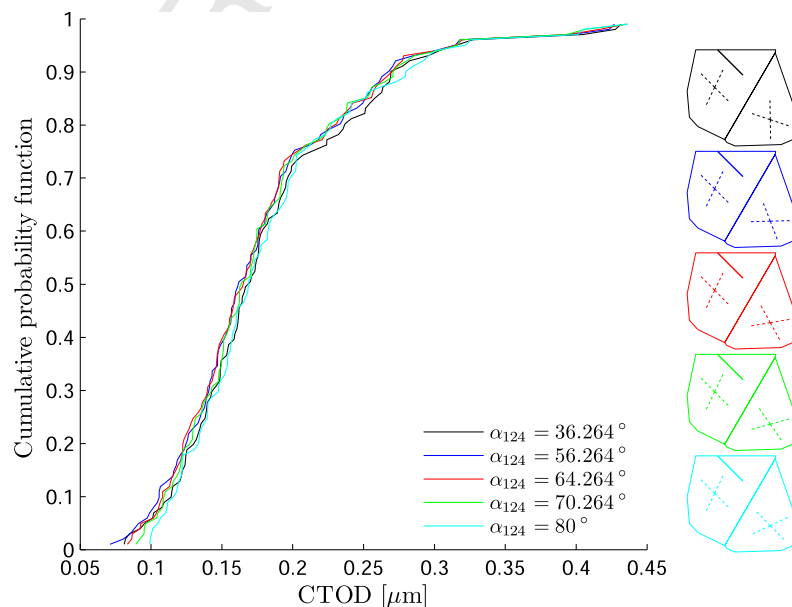


Fig. 12. CTOD histograms for a crack contained in grain 38. Crack length is  $0.5D_{38}$ .



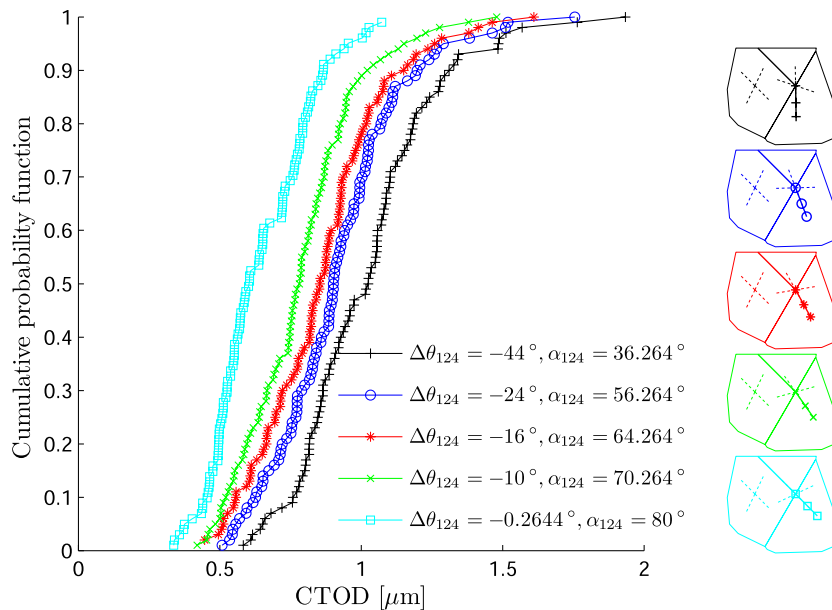


Fig. 13. CTOD histograms for crack in grain 124 and placed in slip plane P4. Crack length in grain 124 is  $0.5D_{124}$ .

381 load. Additionally, the crystallographic orientation also  
 382 affects the stiffness of grain 124. At  $\alpha_{124} = 36.264^\circ$  grain  
 383 124 has the lowest Schmid factors among the analyzed con-  
 384 figurations. However, since the crack in this case is more  
 385 perpendicular to the external load, the CTODs are the  
 386 highest. As we increase the  $\alpha_{124}$ , Schmid values increase,  
 387 but the CTODs decrease since the crack extension moves  
 388 away from being perpendicular to the external load. The  
 389 crack extension direction in this case seems to be the main  
 390 factor influencing the CTOD.

#### 391 4. Conclusions

392 A model of microstructurally short cracks that accounts  
 393 for random grain geometry and crystallographic orienta-  
 394 tions has been coupled with crystal plasticity constitutive  
 395 model. This model is used to study the influence of vari-  
 396 able crack length on crack tip displacements for different  
 397 grain orientations. In all the analyzed cases the crack tip  
 398 displacement increased with larger crack, if the crack is  
 399 contained within the first crack-containing grain.

400 Second crack-containing grain changes the CTOD by val-  
 401 ues by 26% for the case with the shortest crack. As the crack  
 402 is extended into the second crack-containing grain it has to  
 403 change its direction to follow the available slip plane. The  
 404 closer the slip plane is to being perpendicular to the external  
 405 load the larger the CTODs are. Further away the slip plane is  
 406 from this position, lower the CTODs are. In one of these  
 407 cases this resulted in a CTOD decrease of a factor 10.

408 In all analyzed cases significant CTSD values were  
 409 observed. Cracks are therefore of mixed mode loading.

410 The spread of the CTODs due to the random crystallo-  
 411 graphic orientations of grains decreases with increased  
 412 crack length. The ratio between maximal and minimal

CTOD is 4.4 if the crack is contained in the first grain. 413  
 When the crack is extended into the second grain this ratio 414  
 is reduced to still significant 3.3. With further increase of 415  
 the crack length the crack would become less and less 416  
 depended upon the local microstructural features so the 417  
 ratio would continue to decrease. 418

Finally, the model itself needs additional development. 419  
 Primarily, the crystal plasticity material model needs to 420  
 be developed further to account for certain aspects of mate- 421  
 rial cyclic behaviour. At a first stage elastic unloading 422  
 should be implemented whereas in later stages also more 423  
 complex effects such as Bauschinger effect, cyclic hardening 424  
 and softening should be considered. 425

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